On Wick Rotation

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Wick rotation produces numbers that agree with experiment and yet the method is mathematically wrong and not allowed by any self-consistent rule. We explore a small slice of wiggle room in complex analysis and show that it may be possible to use QFT without reliance Wick rotations.

In reference [1] we showed that Bell may have erred in his proof of the non-existence of local hidden variables by using the real number system $\mathbb{R}$. When the derivation is conducted under the hyperreal number system ${}^*\mathbb{R}$, local hidden variables are always allowed [1]. Here we take a similar tack in reanalyzing the rudiments of quantum field theory to show that the conventional way of doing things is not the only possible way. Another way produces a result that might be considered better.

As Zee states in reference [2], "Believe it or not, a significant fraction of the theoretical physics literature consists of varying and elaborating this basic Gaussian integral."

$$G = \int e^{-x^2} dx \tag{1}$$

The purpose of this paper is to explore the idea that the values of such integrals as listed in tables might not be correct.

The rule for complex conjugation is to replace every instance of i with $-i$ so we may write the following.

$$\left(\int e^{-x^2} dx\right)^* = \int e^{-x^2} dx \qquad \Longrightarrow \qquad G^2 = G^* G \tag{2}$$

The normal method to demonstrate the value of G is to square it and convert to plane polar coordinates but that is not the only way. Consider an alternative derivation of the value of G.

$$G^2 = \pi \qquad \Longrightarrow \qquad G = \pm\sqrt{\pi} \tag{3}$$

$$G^* G = \pi \qquad \Longrightarrow \qquad G \in \{\pm\sqrt{\pi}, \pm i\sqrt{\pi}\} \tag{4}$$

In the G^2 version, an appeal is made to the structure of the integral to show that G must be positive so $-\sqrt{\pi}$ is discarded. Likewise, when presented with the G^*G calculation, one might claim that due to the integral form of G, the imaginary answers can also be ruled out. That may well be true but it may also be true that human knowledge of the calculus is as yet incomplete and there is more to be known about G.

Gauss suggested that it is more useful to think of positive, negative, and imaginary numbers as direct, inverse, and lateral respectively. Let the arguments based on the integral structure of G preclude the case of inverse numbers but not lateral ones.

Tables only list the value $\sqrt{\pi}$ and that leads to a situation in which subsequent integrals arising in applications of the least action principle need to be Wick rotated with $t \to -it$ before they can be used to generate numbers for comparison with experiment. Wick rotation is mathematically allowed by physical convention but it destroys the physical interpretation of the equations. If the thing that was time at the beginning of the calculation was written as t, then the thing that appears as it at the end of the calculation should not also be interpreted as time when Wick rotation was the only motivation for its appearance. We motivate the possibility that $G = i\sqrt{\pi}$ by demonstrating that it allows us to make calculations which retain their physical interpretation in the way that is typical of physical calculations: unambiguously and unceasingly.

Consider Zee's further words from reference [2].

> "Integrating by parts under the $\int d^4x$ and not worrying about the possible contribution of boundary terms at infinity..."

The whole thesis of the theory of infinite complexity is that boundary terms at infinity should not be ignored and in reference [3] we showed that time's boundary at infinity in $\int d^4x$ is different than the boundaries on the three spatial dimensions. If each of the three spatial integrations yields a lateral term i as described above, then a factor of $i^3 = -i$ will appear in the integral over dt thus alleviating the need to manually insert it via Wick rotation.

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[1] J. Tooker, ***On Bell's Inequality***, viXra:1312.0173 (2013)

[2] A. Zee, ***Quantum Field Theory in a Nutshell***, 2 Ed, (2010)

[3] J. Tooker, ***Infinitely Complex Topology Changes with Quaternions and Torsion***, viXra:1505.0131 (2015)